Selected Problems from Homeworks:

Homework2:

**Experiment 1**: An experiment was conducted to study the effects of irrigation (Yes or No) and three levels of aerially sprayed pesticide (P1, P2, P3) on the yields of three varieties of corn (V1, V2, V3). There were six fields available for the experiment with two fields randomly assigned to each of the levels of pesticide. Each field was then divided into halves (east-west); one of these halves was randomly assigned to be irrigated, and the other was left without irrigation. Each east-west half of a field was then divided into three regions and the three varieties were randomly assigned to these regions. The number of bushels of corn produced and the total rainfall during the growing season were recorded in each of the regions.

**Answer:** 1. *Type of Randomization:* Complete Randomized Design with a Split-Split Plot Treatment Assignment (2)*Type of Treatment Structure*: 2 × 3 × 3 crossed factorial structure (3.) *Identify each of the factors as being Fixed or Random:* Treatment factors: Irrigation with 2 fixed levels; Pesticide with 3 fixed levels; Variety with 3 fixed levels *4. Describe the experimental units:* EU for Pesticide is Field; EU for Irrigation is Half-Field; EU for Variety is Region in a Half-Field Describe the measurement units: MU is region *5. Response Variable:* number of bushels of corn Covariates: Total rainfall during growing season No subsampling or repeated sampling

**Experiment 2**: A steel manufacturer wanted to study the effect that four sizes of roller gaps, (2 cm, 4 cm, 6 cm, 8 cm), have on a steel manufacturing process with respect to the tensile strength (lbs/in^2 ) of the steel obtained from five different blended alloys (A1, A2, A3, A4, A5). The process consisted of taking a batch of metal from one of the Alloys, dividing the batch into 4 equal portions, and rolling one of the portions with each of the roller gaps. The tensile strength was then determined for the resulting rolled steel. This was repeated for each of the five alloys. The order in which the roller gaps were implemented was randomized for each batch of alloy and the order in which the alloys were run was randomized. Only twenty runs could be accomplished during a single day. The researcher was thus able to observe all five alloys under each of the four roller gaps during a single day. The whole process was repeated on three consecutive days. The carbon content of the alloy may have an effect on the tensile strength of the rolled steel. Therefore, the carbon content of each batch was measured prior to the rolling of the steel.

**Answer:** *1. Type of Randomization*: RCBD with A Split-Plot Treatment Structure *2. Type of Treatment Structure:* 5 × 4 Split Plot Treatment Structure: Whole Plot treatment-Alloy and Split Plot treatment-Roller Gaps *3. Identify each of the factors as being Fixed or Random:* • Blocking Factor: Days with 3 Random levels • Treatment Factors: Whole Plot Factor: Alloys with 5 Fixed levels; Split Plot Factor: Roller Gaps with 4 Fixed levels *4. Describe the experimental units and/or measurement units:* • Whole Plot EU: Batch of Steel; Split Plot EU: Portion of Batch; Describe the measurement units: MU = Portion of Rolled Steel *5. Response Variable:* Tensile strength of steel Covariates: Carbon content of Batch No subsampling or repeated sampling

**Experiment 3:** A microbiologist designed an experiment to evaluate four species of fish on the basis of the amounts of mercury that is transferred to the kidneys of experimental rats that were fed the fish as a part of their diet. The investigators were also interested in comparing three techniques for measuring the amount of mercury in the rats. Five litters, each containing four rats, were randomly selected for use in the experiment. Within each litter, the four rats were randomly assigned to the four species of fish, one species to each rat. After a month of consuming the fish diet, the rats were sacrificed and three sections were randomly selected from the rat’s kidney. The three sections were randomly assigned to the three techniques, and the amount of absorbed mercury was measured in each section using the assigned technique.

**Answer:** *1. Type of Randomization:* • RCBD with Split-plot treatment assignment; blocks are litters *2. Type of Treatment Structure:* • Species crossed with Technique - 4 × 3 Factorial treatment structure *3. Identify each of the Factors as being Fixed or Random:* • Blocking Factor - Litters with 5 Random Levels • Treatment Factors: Species with 4 Fixed Levels; Techniques with 3 Fixed Levels; *4. Describe the Experimental Units and Measurement Units:* • EU for Species is a Rat; EU for Technique is a portion of Rat’s Kidney • MU is a portion of Rat’s Kidney *5. Describe the Measurement Process: Response Variable, Covariates, SubSampling, Repeated Measures* • The response is the percentage of mercury absorbed in the Rat’s kidney. • There are no covariates, subsamples, or repeated measures.

**Experiment 4**: An experiment was designed to compare three different methods of assessing the knowledge obtained by students in an undergraduate statistics course: **• Method 1:** Multiple choice questions • **Method 2:** Student provides detailed solutions to problems • **Method 3:** Individual oral examinations. To conduct the experiment, four sections of STAT 303 will be randomly selected. The four sections are taught by four different instructors. Six students will be randomly selected from each of four different sections of STAT 303. Each student will take all three exams (a total of 72 observations). The researcher is interested in the difference in average scores on the three exams and whether the size of the differences between average scores is consistent across the various sections of STAT 303. There is concern that there may be an effect based on whether a test is taken during the first, second, or third testing period. Hence, each type of testing appears in each testing period. The 6 orders in which the students take the three types of Test Methods are randomly assigned to the students.

**Answer**: *1. Type of Randomization:* • This is a RCBD with blocks being the Sections (or Instructors, in that, Instructor and Section are confounded) and a Crossover Treatment Design *2. Type of Treatment Structure:* • 3 treatments consisting of the three Assessment Methods *3. Identify each of the Factors as being Fixed or Random:* • Blocking Factors: Instructor (or Section) with 4 Random Levels; Period with 3 Fixed Levels; Sequence with 6 Fixed levels; Student with 24 Random levels • Treatment Factor: Assessment Method with 3 Fixed Levels *4. Describe the Experimental Units and Measurement Units:* • EU is Student nested within a Sequence • MU is a Student evaluated with a particular Assessment Method during a particular Period *5. Describe the Measurement Process: Response Variable, Covariates, SubSampling, Repeated Measures* • The response is the score from a student from a particular Assessment Method • There are no covariates or subsamples • Repeated measures are taken on Each student, one during each Assessment Period

Homework3:

**Problem 6.** The Cell Means model, yij = µi + eij has as one of its assumptions that the yij ’s are random, independent observations from the treatment populations.

a. What could the statistician do during the conduct of the experiment in order to ensure that the condition of random, independent observations is reasonable valid?

**Answer:** We have relatively homogeneous experimental units which are randomly assigned to the treatments in order to avoid any subjective assignment of treatments.

A probability sample of units should be selected from available members of each treatment population. Units are selected from within each population such that each unit has an equal chance of entering sample. Note that each population represents a separate treatment classification, and random sampling is maintained only within the population

b. What condition is required of the yij ’s in order for the least squares estimators of µi to be best linear unbiased estimators?

**Answer:** The eij s are uncorrelated with mean 0 and the same variance then the least squares estimators are Best Linear Unbiased Estimators (BLUE) of the population parameters. If we further include the condition that the eij s are independent, normally distributed then the least squares estimators are Uniformly Minimum Variance Unbiased Estimators (UMVUE). Note the difference in these two statements. BLUE’s have smallest variance amongst all unbiased estimators which are linear functions of the data and UMVUE’s have smallest variance amongst all unbiased estimators no matter the form of the estimator.

c. What condition is required of the yij ’s in order to validly use the F-test in AOV hypotheses and to place confidence intervals on the µij ’s?

**Answer:** yij have independent N(µi , σ2 ) distributions for all i and j.

d. Suppose the condition from part c. is not valid. How could you test for differences in the treatment means?

**Answer:** Permutation and rank based procedures would be valid provided we require the condition “yij s are independently distributed with the same variance”. Rank and permutation procedures would not require the normality condition.

***SUMMARY of Procedures*** ***(Handout 4)***

1. Testing a Single Contrast: Use F-test with αC = αF , FamilyWiseErrorRate (FWER). Yields exact result for Pr[Type I Error].

2. Testing a fixed number, M, of contrasts - Selected prior to running the experiment.

(a) If M Contrasts are mutually orthogonal: Use F-test with αC = 1−(1−αE) 1/M. Yields an upper bound of size αF on Pr[Experiment-wise Type I Error].

(b) If M Contrasts are NOT mutually orthogonal: Use F-test with αC = αF /M (Bonferroni F-test) Yields an upper bound of size αF on Pr[Experiment-wise Type I Error].

3. Although it is never a good idea to test contrasts selected after the experiment has been run, the Scheff´e Procedure would be the procedure to be used in the unusual circumstances when this is necessary. Scheff´e’s Procedure handles an unspecified number of contrasts. Scheffe’s Procedure yields Exact Result with Pr[Experiment-wise Type I Error] = αF .

4. For comparing ALL possible Pairs of Treatment Means, µ 0 i s:

(a) Strongly recommend against using Fisher’s Protected LSD and SNK due to the lack of an exact value for FWER

(b) In nearly all cases use Tukey’s HSD when sample sizes are equal.

(c) When the t sample sizes are unequal, use the Tukey-Kramer in place of Tukey.

5. When comparing a Control or Standard Treatment to a Group of Treatments: Use Dunnett’s Procedure.

6. When selecting a subset of Treatments containing the “BEST” Treatment: Use Hsu’s Procedure.

7. When the treatments consist of equally spaced levels of an ordinal scaled variable, use Orthogonal Polynomial with αC = 1 − (1 − αF ) 1/M to evaluate trends in the Treatment Means µ 0 i s. Yields an upper bound of size αF on Pr[Experiment-wise Type I Error].

8. Procedures yielding exact experimentwise error rates are Scheff´e, Tukey’s HSD, and Dunnett’s procedures. Tukey and Dunnett require equal reps for rate to be exact.

9. Procedures yielding results having a bounded experimentwise error rate are Bonferroni and Hsu’s procedures.

10. Fisher’s protected LSD and SNK have experimentwise error rates which are neither exact nor bounded by their nominal values.

11. False Discovery Rate (FDR) is used when the experiment involves a large number of tests of hypotheses. In this situation, the Bonferroni and Scheffe’s Procedure would have a very large probability of a Type II error and many false negatives would occur, that is, failures to determine that the research hypothesis is true.